DECOMPOSITION OF SUPERIMPOSED WAVEFORMS USING THE CROSS TIME FREQUENCY TRANSFORM

P. Bonato, Z. Erim, J.A. Gonzalez-Cueto
NeuroMuscular Research Center, Boston University, Boston, MA USA

Abstract **— The identification of the timing of the discharges of groups of muscle fibers (motor units) is of utmost importance in research into the strategies employed by the central nervous system in producing muscle force and in the diagnosis of neuromuscular diseases. The process involves the recognition of unique shapes (action potentials) contributed by different motor units at random times throughout a muscle contraction. This paper addresses a specific aspect of the identification process: the decomposition of the compound signal when the action potentials of two or more motor units are superimposed. We propose a cross-time-frequency-based procedure to identify which two (out of a previously identified collection of waveforms) are included in a superposition.**

Keywords **— Cross-time-frequency, decomposition, superposition, action potential, motor unit**

I. INTRODUCTION

A variety of algorithms have been proposed in the past to decompose the signal into the basic waveforms related to the firing of different motor units. However, when two or more different action potential waveforms occur at the same time, the automatic decomposition of the signal into the two action potential waveforms is not a trivial task. Traditional filtering approaches are not very successful in this case because motor unit action potential waveforms often partially overlap in frequency. Therefore trivial approaches (i.e. matched filters) are unsatisfactory to solve this problem [8]. The most common approach previously proposed is the peel-off method [4][11] where the superposition is correlated to each individual motor unit template and the best matched template is aligned with the superposition and subtracted from it. The procedure is repeated for the rest of the templates until the energy of the remaining signal is below a preset value. This approach is also unsatisfactory because depending on the delay with which the action potentials overlay each other various peaks can cancel each other out and the resultant signal may not have strong correlation to the individual templates [8]. In order to overcome the limitations of previous approaches we propose to investigate a methodology that relies on a cross-time-frequency based algorithm. The cross-time-frequency representations will be computed using Cohen Class transformations [7] because of the properties of this class of transformations that allow the filtering out of the cross-terms [9][12] that make it difficult to solve the superposition problem.

II. CROSS TIME FREQUENCY BASED DECOMPOSITION OF SUPERPOSITIONS

A. Rationale

The proposed methodology to decompose superpositions will assume that a certain number of unique action potential waveforms, referred to as 'templates', have already been identified. Therefore let x_1, x_2, ... , x_N be the templates derived for *N* motor units and *y* a signal constituted by the superposition of x_i and x_j,which needs to be decomposed. The technique that we propose consists of deriving the cross-time-frequency distribution [6] of each template with the signal *y* constituted by the superposition of any combination of the identified action potential waveforms with unknown delays with respect to each other. It can be shown that when the template utilized to derive the cross-time-frequency distribution is embedded in the signal *y*, the cross-time-frequency distribution - filtered according to criteria designed in the ambiguity domain [10] - is "similar" to the time-frequency distribution of the template. On the contrary, when the template utilized to derive the cross-time-frequency distribution is not embedded in the signal *y*, the filtered cross-time-frequency distribution is not "similar" to the time-frequency distribution of the template.

B. Illustration of the Technique

In the following we illustrate the procedure for a superposition $y = x_1 + x_2$. We first consider the cross-time-frequency representation of x_1 and y, namely $TF_{x_1 y}$, and observe that it can be expressed as

$$TF_{x_1 y} = TF_{x_1 x_1} + TF_{x_1 x_2}$$

where $TF_{x_1 x_1}$ is the time-frequency representation of x_1, and $TF_{x_1 x_2}$ is "half" the interference term [2] of the time-frequency representation of y, TF_{yy} expressed by $TF_{yy} = TF_{x_1 x_1} + TF_{x_2 x_2} + TF_{x_1 x_2} + TF_{x_2 x_1}$. By applying an appropriate kernel to the cross-time-frequency transform $TF_{x_1 y}$, it is possible to reject $TF_{x_1 x_2}$ thus leading to $TF_{x_1 x_1}$.

This is because $TF_{x_1 x_2}$ holds the characteristics of the interference terms [3] and thus, in order to reject it, one may apply the same criteria utilized to attenuate the interference terms which affect the auto-time-frequency representation of a generic signal. In practice an attenuation of $TF_{x_1 x_2}$ will be obtained rather than its complete rejection [12].

The technique is illustrated in Figure 1. The upper plot represents the action potential waveforms of two templates, $x_1(t)$ and $x_2(t)$. The plot in the middle shows the time-frequency representation of the two templates. The lower plot displays the cross-time-frequency representation of $x_1(t)$ and $y(t)=x_1(t)+x_2(t)$

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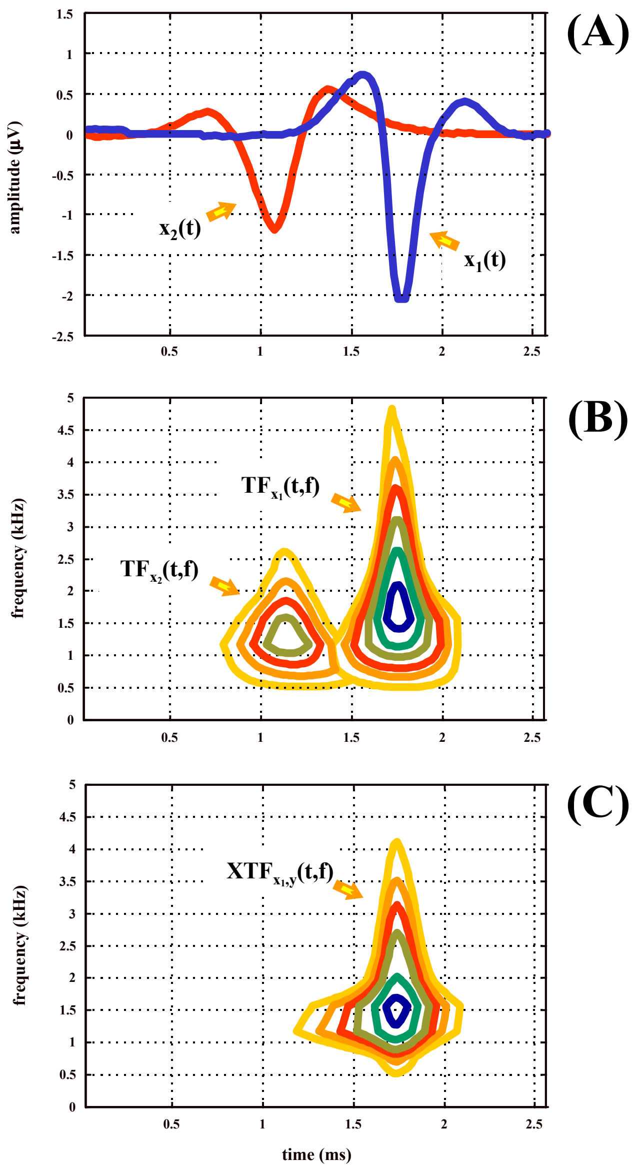

Figure 1. The cross-time-frequency based technique to solve the superposition of waveforms**.** (A) Superimposed waveforms $x_1(t)$ and $x_2(t)$. (B) Time-frequency distributions TF_{x_1} and TF_{x_2}. (C) The Choi-Williams cross-time-frequency distribution computed using the first template $x_1(t)$ with the superimposed signal $y(t)=x_1(t)+x_2(t)$.

obtained using the Choi-Williams transformation [5]. The cross-time-frequency representation in Figure 1 is apparently "similar" to the time-frequency representation of $x_1(t)$.

C. *Interpretation in the Ambiguity Domain*

To further demonstrate how the cross time-frequency approach suppresses undesirable terms and 'teases out' the template embedded in the superposition, consider the cross-ambiguity function derived by x_1 and y

$$AF_{x_1 y} = AF_{x_1 x_1} + AF_{x_1 x_2} \qquad (1)$$

where AF indicates either the ambiguity function or the cross-ambiguity function. Subscripts correspond to those defined above for the time-frequency representations. It is well known that the term $AF_{x_1 x_1}$ is usually concentrated over the axes of the ambiguity plane, while the term $AF_{x_1 x_2}$, which corresponds to a "half" interference term, is expected to be away from the axes. The distance of $AF_{x_1 x_2}$ from the origin of the ambiguity plane is a function of the distance between the baricenters of the auto-time-frequency representations of x_1 ($TF_{x_1 x_1}$) and x_2 ($TF_{x_2 x_2}$). Therefore kernels which attenuate components of the ambiguity function that are away from the axes lead to a reduction of the amplitude of $TF_{x_1 x_2}$.

Figure 2 shows the ambiguity functions of the action potential waveforms represented in Figure 1. The ambiguity function of $x_1(t)$ and the cross-ambiguity function of $x_1(t)$ and $x_2(t)$ are displayed. The sum of these two functions is equal to the cross-ambiguity function of $x_1(t)$ and $y(t)=x_1(t)+x_2(t)$, as seen from equation (1) above. The coordinates of the baricenter of the cross-ambiguity function of $x_1(t)$ and $x_2(t)$ are equal to the distance in time and frequency of the time-frequency representations of the two components [9]. This figure also emphasizes the relevance of an appropriate choice of the filtering function in the ambiguity domain. The optimal filter would maximize the attenuation of the cross-ambiguity function of $x_1(t)$, $x_2(t)$ and not alter the ambiguity function of $x_1(t)$. Possible design criteria are discussed in the last section of this paper.

D. *Classification Procedure*

With $y = x_i + x_j$ and i and j being unknown, the proposed classification procedure requires the computation of all the cross-time-frequency representations of x_k (with k=1, ... , N)

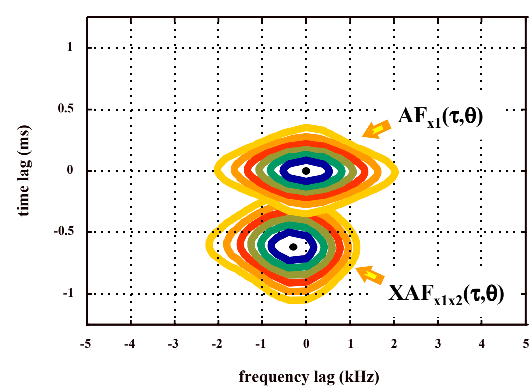

Figure 2. The ambiguity function of the template $x_1(t)$ and cross-ambiguity function of $x_1(t)$ and $x_2(t)$. The relationship between the ambiguity and cross-ambiguity functions are used to filter the cross-ambiguity function in order to identify the template contributing to the superposition.

and y thus

$$TF_{x_k y} = TF_{x_k x_i} + TF_{x_k x_j}$$

and the cross-ambiguity function

$$AF_{x_k y} = AF_{x_k x_1} + AF_{x_k x_2} .$$

It is apparent that in cases where $j \neq i$ *or* j, the cross-ambiguity function is expected to be located away from the axes while the energy around the origin of the ambiguity plane is expected to be negligible. In fact, in these cases, we are computing the cross-ambiguity function of cross-products among different components, i.e. templates, thus leading to terms that are like the interference terms.

An example is shown in Figure 3 that shows the output of the proposed procedure when one computes the cross-time-frequency representation of the superimposed waveform $y(t)=x_1(t)+x_2(t)$ with a template $x_3(t)$. The upper plot of Figure 3 shows the motor unit action potential waveforms $x_1(t)$ and $x_2(t)$, that contribute to the superimposed waveform, together with a third template $x_3(t)$. The plot in the middle represents the time-frequency distribution of the template $x_3(t)$. The lower plot reports the cross-time-frequency distribution of $x_3(t)$ and $y(t)=x_1(t)+x_2(t)$ computed using a Choi-Williams transformation [5]. Clearly, the derived cross-time-frequency distribution does not even slightly resemble the time-frequency distribution of $x_1(t)$.

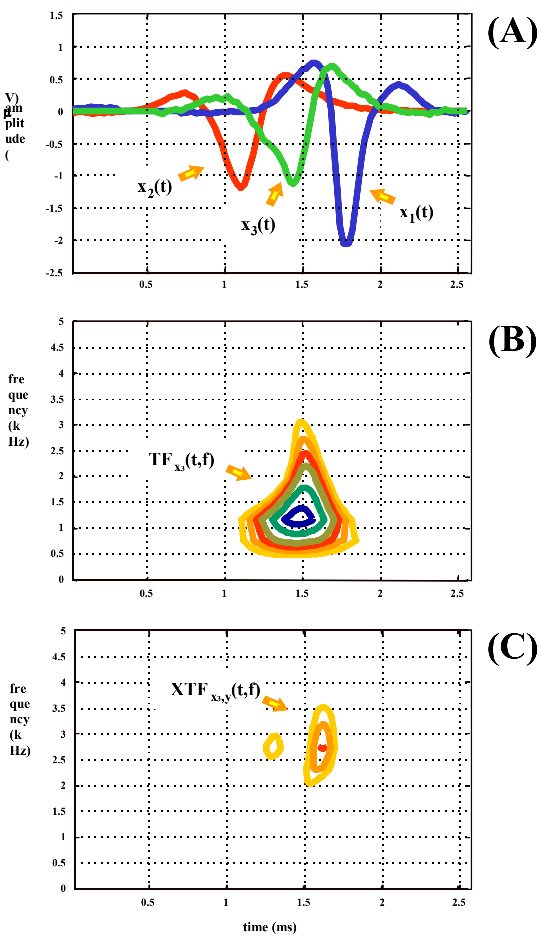

Figure 3. The rejection of a template that is not included in a superposition. (A) Two templates $x_1(t)$ and $x_2(t)$, that contribute to a superposition $y(t)= x_1(t) + x_2(t)$, and a third template $x_3(t)$ that is not included in the superposition. (B) Time-frequency representation of the third template $x_3(t)$. (C) The Choi-Williams cross-time-frequency distribution of $y(t)= x_1(t) + x_2(t)$ and $x_3(t)$.

Figures 1 and 3 summarize the rationale for the identification procedure. When one computes the cross-time-frequency distribution of the superimposed waveform and a template that does contribute to the superposition, the computed representation on the time-frequency domain is "similar" to the distribution of the template. Conversely, when one derives the cross-time-frequency distribution of the superimposed waveform and a template that does not contribute to the superposition, the derived representation on the time-frequency domain does not even resemble the time-frequency distribution of the template. Therefore, using a measure of "similarity", i.e. distance, between the cross-time-frequency representation and the time-frequency distribution of the template, one may define a way to identify the templates that contribute to the superimposed waveform.

E. Filtering of the Cross Ambiguity Function

To increase the "similarity" between time-frequency and cross-time-frequency representations (Figure 2) we compared different approaches to filtering the cross-ambiguity function (i.e., choices of the kernel of the cross-time-frequency transformation). Specifically, we utilized the following transformations: 1) cross-Wigner-Ville (XWV); 2) Choi-Williams (CW) (σ=0.1) [10]; 3) Radially Gaussian Kernel (RGK) [1]; 4) a transformation based on a modified design of the RGK kernel (Modified RGK), derived by utilizing the entire ambiguity function domain for the convergence of the algorithm as opposed to the first and second quadrants; 5) a Matched Kernel transformation designed, for each template under consideration, as the square of the magnitude of its ambiguity function. Figure 4 displays the improvement introduced by the use of the Matched Kernel.

F. Simulations

Five actual action potential waveforms (templates) recorded during a muscle contraction were used to simulate superimposed waveforms. A total of 400 superpositions were obtained by combining pairs of templates with different degrees of overlap, i.e., shifting one template with respect to the other (range ±1 ms). For each superposition, the cross-time-frequency representations of the superposition and each of the templates were computed using the methods described above. Then the Kolmogorov distance between each cross-time-frequency representation and the time-frequency distribution of the template used to derive the cross-time-frequency representation was computed. The superposition was considered to be made up of the two templates that resulted in the lowest two distances.

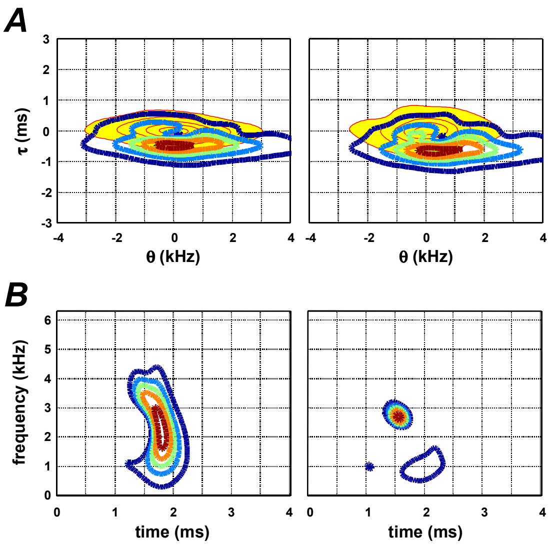

Figure 4. Improvements afforded by filtering in the ambiguity domain. A. The cross-ambiguity functions between the waveforms x_1 and x_3 with the superposition waveform $y = x_1 + x_2$ (see Figure 2). The kernels are shown as shaded areas. B. The resultant cross-time-frequency transforms between the superposition waveform and the templates x_1 and x_3. Note the increased similarity between the cross-time-frequency representation and the time-frequency distribution of x_1 included in the superposition.

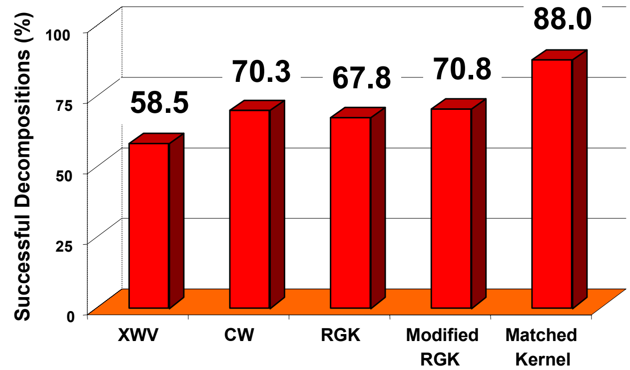

Figure 5. Successful decompositions of the proposed procedure. The different bars show the results for the cross-Wigner-Ville transform (XWV), the cross-Choi-Williams cross-transformation (CW), the radially Gaussian kernel cross-distribution (RGK), its modified version (Modified RGK), and the Matched Kernel approach.

III. PRELIMINARY RESULTS

Our study indicates that the proposed method can identify the motor unit action potential waveforms contributing to a superposition with reasonable success. Marked differences were observed when different filtering approaches were applied to the cross-ambiguity functions. The XWV approach resulted in the lowest performance, i.e., 58.5 % of the decompositions were successful. The Matched Kernel approach was the best (88 % successful decompositions).

IV. CONCLUSIONS

A method to solve the superposition of action potential waveforms has been proposed based on the use of cross-time-frequency transformations. A preliminary study to assess its application to real data indicates the suitability of the technique. However, future work needs to be done in order to fully characterize the methodology. Specifically, a point of paramount importance is the characterization of the methodology when the motor unit action potential waveforms slightly change in time, as it happens because of the physiology of the muscle contraction.

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